Variable selection based on entropic criterion and its application to the debris-flow triggering

Chien-chih Chen[1], Chih-Yuan Tseng[2] and Jia-Jyun Dong[3]
[1] Institute of Geophysics, [2] Department of Physics, and
[3] Institute of Applied Geology
National Central University, Jhongli, Taiwan 320

Abstract

We propose a new data analyzing scheme, the method of minimum entropy analysis (MEA), in this paper. New MEA provides a quantitative criterion to select relevant variables for modeling the physical system interested. Such method can be easily extended to various geophysical/geological data analysis, where many relevant or irrelevant available measurements may obscure the understanding of the highly complicated physical system like the triggering of debris-flows. After demonstrating and testing the MEA method, we apply this method to a dataset of debris-flow occurrences in Taiwan and successfully find out three relevant variables, i.e. the hydrological form factor, numbers and areas of landslides, to the triggering of observed debris-flow events due to the 1996 Typhoon Herb.

1 Introduction

Most geophysical/geological problems, e.g. the trigging of debris-flows or earthquakes, are so complicated that many observed and/or unobserved variables have their obscure contributions to the geophysical/geological systems [1]. From the viewpoint of practical experiment setting, scientists quite often encounter a problem of variable selection to choose relevant measurement regarding their own physical systems. For instance, three categories of variables describing three aspects of topography, geology and hydrology, are usually used in the geographic information system (GIS) to assess the hazard potential of debris-flows (e.g. [2]; [3]). Although some consensus could be reached for the problem of debris-flow trigging, the variables measured could be much different among different research groups [4]. Therefore, when lots of measurement could be probably made and available to use, we are forced to face a fundamental question of which variables are relevant to describe a highly complex physical system like the debris-flow system.

To answer the abovementioned question, we in this study introduce a new data analyzing scheme, i.e. the minimum entropy criterion ([5]; [6]), to the problem of selecting the variables which dominate the debris-flow occurrence. We first present the principle of minimum entropy analysis (MEA) and verify its result when applying to a geological example extracted from the textbook of Davis [7]. Then, in Sec. 3, we demonstrate the application of MEA to an observed debris-flow dataset [8], consisting of the binary outcome (the response) of debris-flow occurrences and measurement (the covariates) of some topographic, geologic and hydrologic variables. Conclusion will be given at the end of this paper.

2 Minimum entropy rule for variable selection

2.1 Principle of minimum entropy analysis

Model selection in data processing is usually achieved by ranking models according to the increasing order of *preference*. Several methods such as P-values, Bayesian approaches and Kullback-Leibler distance method, etc., are some popular examples to provide pertinent selection criteria ([5]; [9]; [10]; [11]; [12]). Tseng [5] reviews those methods and suggests an entropy-based criterion as the selecting preference of models.

Principle of maximum entropy proposed by Jaynes ([13], [14], [15]) is recognized as a tool to assign a probability distribution to a system. This tool involves the use of a unique functional form of entropy $S\left[P\right] = -\sum_x P\left(x\right)\ln P\left(x\right)$, where x denotes states of the model and P is the probability density function. It is uniquely determined through Shannon's axiomatic approach. Since Jaynes's work, this tool was further extended to become an inductive inference tool for information processing, for updating the probability distribution of a system according to information in hand ([16]; [17]). Similarly, when a reference density function $m(x)$ is available, one can also show that the *relative* entropy involves another unique functional form, $S\left[P\left|m\right.\right] = -\sum_x P\left(x\right)\ln P\left(x\right)/m\left(x\right)$ [5].

Tseng [5] has shown that the relative entropy uniquely determines the preference for model selection. Suppose a family of models is given by probability distributions $\{P^m\left(x\right)\}$, where m labels the model. The preference given by the relative entropy of model $P^m\left(x\right)$ and a reference measure $\mu\left(x\right)$,

$$S\left[P^m\left|\mu\right.\right] = -\sum_x P^m\left(x\right)\ln P^m\left(x\right)/\mu\left(x\right) \ , \tag{1}$$

is a scalar value. Such scalar relative entropy measures differences between model $P^m\left(x\right)$ and a reference measure $\mu\left(x\right)$ [5]. Maximizing the relative entropy $S\left[P^m\left|\mu\right.\right]$ indicates $P^m\left(x\right)$ to equal to the reference measure $\mu\left(x\right)$.

If the reference measure is chosen to be the distribution $P_{\text{real}}\left(x\right)$ that is believed to be able to interpret the system correctly, maximizing $S\left[P^m\left|P_{\text{real}}\right.\right]$ indicates that the model $P^m\left(x\right)$ is the most preferable. Unfortunately, the real distribution is usually difficult to be practically determined. Tseng [5] proposes to rank models according to the relative entropy $S\left[P^m\left|\mu\right.\right]$ with the reference measure $\mu\left(x\right)$ being set to a uniform probability distribution $P_{\text{uni}}\left(x\right)$. Since a uniform distribution does not carry any information about the system, maximum $S\left[P^m\left|P_{\text{uni}}\right.\right]$ indicates that $P^m\left(x\right)$ is identical to P_{uni} and the model $P^m\left(x\right)$ carries no information about the system at all. On the other hand, when a model $P^m\left(x\right)$ is codified with more information, $P^m\left(x\right)$ differs from the uniform distribution more. Thus, decreasing the relative entropy $S\left[P^m\left|P_{\text{uni}}\right.\right]$ should provides same preference of different models given by increasing $S\left[P^m\left|P_{\text{real}}\right.\right]$.

In the case of variable selection, let's suppose a regression model $P\left(\overrightarrow{x}\right)$ associated with N variables $\overrightarrow{x} = \{x_1, x_2, \cdots x_N\}$ is given to reveal the behavior of an unknown system from experimental measurements. For example, the logit model is often used for a system with the binary outcome ([18]; [12]; [3]). Note that those variables $\overrightarrow{x}$ are usually assessed according to experiments (observations) and may or may not denote crucial characteristics of the system interested. Besides, they may be correlated to each other. Our question, then, is that, after modeling an unknown system with different combinations of variables, which ones play the most important roles allowing the model to pertinently interpret the system. Namely, what is the preference of those variables? This is basically the same question addressed in model selection by Tseng [5].

Suppose that a full model defined by $P_{\text{full}}\left(\overrightarrow{x}\right)$ is the model containing all N variables available from experiments. Since given a set of N variables, there will be 2N - 2 combinations (subsets) of variables $\overrightarrow{x}_{s_i} \in \overrightarrow{x}$. Each subset forms a submodel $P_{s_i}\left(\overrightarrow{x}_{s_i}\right)$. According to Eq. (1) with $P^m\left(x\right)$ being replaced by $P_{s_i}\left(\overrightarrow{x}_{s_i}\right)$ and $\mu\left(x\right)$ being given by a uniform distribution, increasing ranking

order of the preference for these submodels is given by decreasing the relative entropy

$$S\left[P_{s_i}\left|P_{\mathrm{uni}}\right.\right] = -\sum_{\overrightarrow{x}_{s_i}\in\overrightarrow{x}} P_{s_i}\left(\overrightarrow{x}_{s_i}\right)\ln\frac{P_{s_i}\left(\overrightarrow{x}_{s_i}\right)}{P_{\mathrm{uni}}} = S\left[P_{s_i}\right] + \ln P_{\mathrm{uni}}\ , \tag{2}$$

where the submode $P_{s_i}\left(\overrightarrow{x}_{s_i}\right)$l contains n_i variables and $S\left[P_{s_i}\right] = -\sum_{\overrightarrow{x}_{s_i}\in\overrightarrow{x}} P_{s_i}\left(\overrightarrow{x}_{s_i}\right)\ln P_{s_i}\left(\overrightarrow{x}_{s_i}\right)$. Since $\ln P_{\mathrm{uni}}$ is a constant, ranking order given by decreasing $S\left[P_{s_i}\left|P_{\mathrm{uni}}\right.\right]$ is identical to that one from decreasing $S\left[P_{s_i}\right]$. After determining the ranking preference of submodels, the selection of variables then can be made from analysis of those submodels thus ranked. The detailed process will be illustrated in the following.

2.2 Demonstration and verification of minimum entropy analysis

We test our data processing procedure of the MEA method with an example of sample classification extracted from the book of [7]. Table 1 contains the results of brine analyses for oil-field waters from three groups of carbonate units in Texas and Oklahoma. Brines recovered during drillstem tests of wells may have relict compositional characteristics that provide clues to the origin or depositional environment of their source rocks. The first column in Table 1 denotes the brine samples belonging or not belonging to some specific carbonate unit (Grayburg Dolomite, briefly in "Unit G" here), while the rest are the percentages of six chemical ions. Davis [7] applies the discriminant function analysis (DFA) to these six multivariate measurements for finding a projection, i.e. a linear combinations of measurements, allowing various categories of samples to be distinguished. The first discriminant function thus calculated is (-0.3765, -0.0468, 0.0112, -0.0148, -0.0174, -0.0110)·(HCO_3, SO_4, Cl, Ca, Mg, Na)$^{\mathrm{T}}$, which can clearly separates samples from Unit G and other units. Note that the weighting factors in the first discriminant function for variables of HCO_3 and SO_4, i.e. -0.3765 and -0.0468, represent the first two largest factors in magnitude among six, thus indicating these two variables play the most dominant effect in classification.

Table I: Chemical analyses of brines (in ppm) recovered from drillstem tests of three carbonate rock units (Ellenburger Dolomite, Grayburg Dolomite = Unit G, Viola Limestone) in Texas and Oklahoma. Adapted from Davis [7].

Unit G	HCO_3	SO_4	Cl	Ca	Mg	Na
N	10.4	30	967.1	95.9	53.7	857.7
N	6.2	29.6	1174.9	111.7	43.9	1054.7
N	2.1	11.4	2387.1	348.3	119.3	1932.4
N	8.5	22.5	2186.1	339.6	73.6	1803.4
N	6.7	32.8	2015.5	287.6	75.1	1691.8
N	3.8	18.9	2175.8	340.4	63.8	1793.9
N	1.5	16.5	2367	412	95.8	1872.5
Y	25.6	0	134.7	12.7	7.1	134.7
Y	12	104.6	3163.8	95.6	90.1	3093.9
Y	9	104	1342.6	104.9	160.2	1190.1
Y	13.7	103.3	2151.6	103.7	70	2054.6
Y	16.6	92.3	905.1	91.5	50.9	871.4
Y	14.1	80.1	554.8	118.9	62.3	472.4
N	1.3	10.4	3399.5	532.3	235.6	2642.5
N	3.6	5.2	974.5	147.5	69	768.1
N	0.8	9.8	1430.2	295.7	118.4	1027.1
N	1.8	25.6	183.2	35.4	13.5	161.5
N	8.8	3.4	289.9	32.8	22.4	225.2
N	6.3	16.7	360.9	41.9	24	318.1

Can we identify relevant variables in the problem of determining the category of samples in Table 1, by means of our entropy-based procedure?

Let's consider the response to be the binary outcome belonging ("Y" or "1") or not belonging ("N" or "0") to Unit G and the covariates those percentages of six chemical ions in Table 1. We can apply the logit model ([12]; [18])

$$R\left(\overrightarrow{x}\right) = \frac{\exp\sum_{i=1}^{N}\beta_i x_i}{\exp\sum_{i=1}^{N}\beta_i x_i + 1} \tag{3}$$

to relate the response to the covariates. Normalizing Eq. (3), probability distribution of the response for a given subset of all 6 variables is

$$P\left(\overrightarrow{x}\right) = R\left(\overrightarrow{x}\right)/Z = \frac{1}{Z}\frac{\exp\sum_{i=1}^{N}\beta_i x_i}{\exp\sum_{i=1}^{N}\beta_i x_i + 1} \tag{4}$$

where $Z = \sum_{\overrightarrow{x}} \frac{\exp\sum_{i=1}^{N}\beta_i x_i}{\exp\sum_{i=1}^{N}\beta_i x_i + 1}$ is the normalization constant. Note that coefficients β_i could be determined through fitting the logit model to experimental measurements by the maximum likelihood estimation [18]. Thus, the entropy of $P\left(\overrightarrow{x}_{s_i}\right)$, i.e. Eq. (2), with different subsets of variables $\overrightarrow{x}_{s_i} \in \overrightarrow{x}$ gives the ranking order of different submodels $P\left(\overrightarrow{x}_{s_i}\right)$ defined by Eq. (4) with $\overrightarrow{x}$ being replaced by $\overrightarrow{x}_{s_i}$.

In the example of brine data there are 62 submodels. We found 16 submodels among those 62 to have the minimum entropy value of ˜1.7918 as shown in Table 2, while the rest of the submodels have the entropy larger than 2. The MEA suggests that these 16 submodels to be the most preferable. Yet due to the intrinsically finite precision of measured data, we can not distinguish the preferences of these 16 submodels further. Tackling the issue of entropy resolution resulted from the intrinsic measurement precision there are many possible ways (e.g. [12]) to determine the most dominate variables in this example. Here we simply count the frequencies of six variables appeared in these 16 submodels. It turns out that the frequencies for variables of HCO_3 and SO_4 are 16 and 15, respectively, and 8 for the rest of variables. This result suggests that the ability of interpreting the experimental measurements by the logit model is strongly dominated by simultaneously associating variables of HCO_3 and SO_4 in the data. And, such result is much consistent with the DFA. Comparing both results from the DFA and the MEA procedures improves the understanding and enhances the confidence in our entropy-based technique.

3 Application of minimum entropy analysis to the debris-flow trigging

Taiwan located at an active convergent plate boundary is an island with rugged topography and severe erosion. During heavy rainfalls brought by typhoons, the occurrence of debris-flows often results in enormous damage of life and buildings ([2], [8], [19], [20], and [21]). There are absolutely many factors intricately affecting the occurrence of the debris-flows and various field measurement has been conducted to assess the occurrence potential of debris-flows in Taiwan ([2], [4], [8], [19], [21], [22], and [23]). So, can we figure out the observations relevant to the trigging of debris-flows by means of our MEA procedure? To preliminarily apply the MEA method, we have used a relatively small dataset documenting the occurrence of debris-flows (Table 3) in the Hsinyi area of Nantou County, Central Taiwan, during the 1996 Typhoon Herb [8]. The related variables including gully lengths (Le), areas of drainage basin with slope $> 15^o$ (Ad), form factor (Ff = Ad/Le^2), and numbers (Nl) and areas (Al) of landslides, which implicitly reflect the topographic, geologic and hydrologic characteristics of examined gullies, are listed in Table 3. For the detailed description of field observations, please refer to the paper of Lin et al. [8].

Table II: Entropy (S) for sixteen submodels with different combinations of six variables in Table 1 (A = HCO3, B = SO4, C = Cl, D = Ca, E = Mg, and F = Na). "1" or "0" denotes the variable selected or not selected in each submodel.

A	B	C	D	E	F	S
1	1	1	0	1	1	1.79183823
1	1	1	1	0	1	1.79183829
1	1	0	1	1	1	1.79183836
1	1	1	1	1	0	1.79183836
1	1	0	1	1	0	1.79184075
1	1	0	0	1	1	1.79184177
1	1	1	0	1	0	1.79184215
1	1	1	0	0	1	1.79184241
1	1	0	0	1	0	1.79184396
1	1	0	1	0	1	1.79184653
1	1	1	1	0	0	1.79184701
1	0	1	1	1	1	1.79184888
1	1	0	1	0	0	1.79184968
1	1	1	0	0	0	1.79185471
1	1	0	0	0	1	1.79185668
1	1	0	0	0	0	1.79185738

Rupert et al. [3] used a logistic regression to predict the probability of the debris-flow occurrence, and their results show that the logistic regression is a valuable tool in the debris-flow prediction. Therefore we utilize the logit model, again, to relate the binary outcome of the debris-flows to five covariates listed in the last five columns in Table 3. Following the processing procedure demonstrated in Sec. 2.2, we out of 30 submodels found 3 submodels having the minimum entropy of ˜2.93 as shown in Table 4, i.e. Models 1, 2 and 3. The dilemma of entropy resolution also appears in this case. Two approaches are useful in determining the model(s) with the minimum entropy. We have listed in Table 4 all the calculated entropy of 30 submodels for the debris-flow data. The calculated entropy of 30 submodels ranges between 2.9346 and 3.0726, and the difference in entropy is about 0.138. When the resolution level in entropy is assigned 10% which is expected to be related to the measured precision in observation Model 4 with the entropy of 2.9538 could then be discriminated from the first 3 models with the entropy of ˜2.93, because the entropy difference between Model 4 and the first 3 models is larger than 10% of 0.138. On the other hand, according to the debris-flow data shown in Table 3, it seems fairly conservative to consider the measurement precision is with three significant digits. Therefore, the significant figure in entropy is also down to the second digits after the decimal point and we still conclude the first 3 models with the minimum entropy of ˜2.93 are the most preferable.

Same three variables of form factor (Column C in Table 4), numbers (Column D in Table 4) and areas (Column E in Table 4) of landslides are incorporated into all the 3 submodels with the entropy of ˜2.93, meaning that those three variables are important to the debris-flow triggering, particularly, in the studied areas of the dataset we used. We have noticed that, in the two papers of [8] and [2], the same watershed was studied and both the observation spans of debris-flows are after the 1996 Typhoon Herb. Therefore it is quite interesting to compare our MEA result with the assessing variables used by the expert system in [2]. In Lin et al. [2] an overall debris-flow hazard index is derived from a sophisticated GIS analysis of nine factors, i.e. rock formation, fault length, landslide area, slope angle, slope aspect, stream slope, watershed area, form factor and C factor (for the detailed explanation of these factors, please refer to their paper). The data we used, as mentioned above, only represents a relatively small dataset. However, two variables of form factor and landslide area are agreeably selected to be the important factors for the debris-flow triggering in both our

Table III: Debris-flow occurrences of 22 creeks (the 1st column) during the 1996 Typhoon Herb in Hsin-Yi area of the Nantou County, Central Taiwan, together with their corresponding characteristics including gully lengths (Le), areas of drainage basin with slope $> 15^o$ (Ad), form factor (Ff = Ad/Le^2) and numbers (Nl) and areas (Al) of landslides. Adapted from Lin et al. [2].

Occurrence	Le [m]	Ad [km^2]	Ff	Nl	Al [km^2]
No	1505	0.86	0.3797	0	0
Yes	1876	1.27	0.3609	3	10.9
Yes	1640	0.35	0.1301	2	4.5
Yes	1560	0.57	0.2342	3	3.4
Yes	2158	1.82	0.3908	5	3.7
Yes	1035	1.82	1.6990	1	7.8
Yes	582	3.4	10.0377	5	3.7
Yes	2445	3.4	0.5687	4	4.4
Yes	2685	3.5	0.4855	9	8.4
No	2350	1.97	0.3567	0	0
No	142	1.15	57.0323	4	2.7
No	1349	1.55	0.8517	4	2.9
No	1337	0.74	0.4140	4	1.2
No	911	0.72	0.8676	3	0.8
Yes	2048	0.78	0.1860	5	0.086
Yes	2960	2.18	0.2488	6	0.226
No	2010	1.65	0.4084	6	0.061
Yes	675	0.58	1.2730	1	0.033
Yes	4947	2.24	0.0915	13	0.362
No	3185	4.05	0.3992	4	0.045
Yes	4209	6.63	0.3742	7	0.084
Yes	4444	6.93	0.3509	21	0.371

MEA result and the GIS analysis of Lin et al. [2], indicating a fairly good performance of the MEA procedure. One important fact is that our MEA procedure obviously provides a quantitative criterion in variable selection for the debris-flow triggering while the reason for the selection of those nine factors in Lin et al. [2], as they mentioned, is quite subjective and based primarily on individual opinion and experience.

Then, the MEA procedure raises an open issue about whether the variable of landslide number really does matter to the triggering of debris-flows. We postpone to future work the examination of this issue.

4 Concluding remark

In the data analysis, two questions are commonly addressed. What is the pertinent model to best interpret experimental measurements for understanding the physical system interested? And, what are the most important variables that should be employed in the model? One may be able to reveal natures and properties of the system through answering these two questions. For the first question, unfortunately, there is no systematical method to answer it. It is usually resolved through the methods of trials and errors, empirical regressions, and some intuitive assumptions etc. We therefore focus on answering the second question here. Our proposed MEA procedure represents a systematical scheme to tackle this fundamental issue. We establish, demonstrate and test our MEA procedure by two geoscientific examples in this paper. The MEA procedure can then be satisfactory to provide a quantitative criterion to the selection of relevant variables in both examples.

To the course of data analysis, the MEA procedure seems simple and straightforward. It is thus

Table IV: Entropy (S) for total thirty submodels with different combinations of five variables in Table 3 (A = Le, B = Ad, C = Ff, D = Nl, and E = Al). "1" or "0" denotes the variable selected or not selected in each submodel.

Submodel No.	A	B	C	D	E	S
1	0	1	1	1	1	2.9346
2	1	0	1	1	1	2.9351
3	0	0	1	1	1	2.9355
4	1	1	1	0	1	2.9538
5	1	1	0	1	1	2.9542
6	1	0	1	0	1	2.9542
7	1	0	0	1	1	2.9557
8	1	0	0	0	1	2.9649
9	1	1	0	0	1	2.9653
10	0	1	0	1	1	2.9725
11	0	0	0	1	1	2.9738
12	0	1	1	0	1	2.9747
13	0	0	1	0	1	3.0065
14	0	1	0	0	1	3.0117
15	1	1	1	1	0	3.0240
16	1	0	1	1	0	3.0279
17	0	1	1	1	0	3.0289
18	0	0	1	1	0	3.0296
19	1	1	1	0	0	3.0446
20	0	1	1	0	0	3.0447
21	0	0	0	0	1	3.0447
22	1	0	1	0	0	3.0498
23	1	1	0	1	0	3.0550
24	1	0	0	1	0	3.0550
25	0	1	0	1	0	3.0560
26	0	0	0	1	0	3.0570
27	0	0	1	0	0	3.0607
28	1	1	0	0	0	3.0632
29	1	0	0	0	0	3.0638
30	0	1	0	0	0	3.0726

expected that the MEA procedure could be easily extended to various geophysical/geological data analysis, where many relevant or irrelevant possible measurements could obscure the understanding of the highly complicated physical system. The triggering of debris-flows is such an example. We would also like to emphasize here that the MEA procedure only provides an honest way to extract the most effective information from dazzling variables in hand. It can not guarantee the precision and the correctness of measurement, which means the datum itself could be incorrect and the measurement could be conducted in ill condition. This should be a general property for all the data analyzing techniques.

Acknowledgment

CCC is grateful for research support from both the National Science Council (ROC) and the Institute of Geophysics (NCU, ROC). Research by CYT is grateful for research support from the National Science Council (ROC).

References

[1] Rundle, J.B., Klein, W., Tiampo, K.F., and Gross, S.J., 2000. Linear pattern dynamics in nonlinear threshold systems, Phys. Rev. E, 61(3), 2418-2431.

[2] Lin, P.S., Lin, J.Y., Hung, J.C., and Yang, M.D., 2002. Assessing debris-flow hazard in a watershed in Taiwan, Engineering Geology, 66, 295-313.

[3] Rupert, M. G., Cannon, S.H., and Gartner, J.E., 2003. Using Logistic Regression to Predict the Probability of Debris Flows Occurring in Areas Recently Burned by Wildland Fires, Open-File Report OF 03-500, US Geological Survey.

[4] Wieczorek, G.F., and Naeser, N.D., 2000. Debris-Flow Hazards Mitigation: Mechanics, Prediction, and Assessment, A.A. Balkema, Rotterdam, Netherlands.

[5] Tseng, C.Y., 2006. Entropic criterion for model selection, Physica A, 370, 530-538.

[6] Tseng, C.Y., and Chen, C.C., 2006. On variable selection and data analysis: the minimum entropy analysis, Phys. Rev. E (submitted).

[7] Davis, J.C., 2002. Statistics and Data Analysis in Geology, 3rd. Ed., Wiley, New York.

[8] Lin, C.W., Wu, M.C., Shieh, C.L., and Shieh, Y.C., 2000. Influence of geology on debris-flows: Examples from Hsin-Yi, Nantou County, Taiwan, in Debris-Flow Hazards Mitigation: Mechanics, Prediction, and Assessment, G.F. Wieczorek and N.D. Naeser (Eds.), 169-176, A.A. Balkema, Rotterdam, Netherlands.

[9] Weiss, R.E., 1995. The influence of variable selection: a Bayesian diagnostic perspective, J. Am. Stat. Assoc., 90, 619-625.

[10] Raftery, A.E., Madigan, D., and Hoeting, J.A., 1997. Bayesian model averaging for regression models, J. Am. Stat. Assoc., 92, 179-191.

[11] Forbes, F., and Peyrard, N., 2003. Hidden Markov random field model selection criteria based on mean field-like approximations, IEEE Trans. Pattern Anal. Mach. Intell., 25(9), 1089-1101.

[12] Dupuis, J.A., and Robert, C.P., 2003. Variable selection in qualitative models via an entropic explanatory power, J. Stat. Plann. Infer., 111, 77-94.

[13] Jaynes, E.T., 1957a. Information theory and statistical mechanics, Phys. Rev., 106, 620-630.

[14] Jaynes, E.T., 1957b. Information theory and statistical mechanics II, Phys. Rev., 108, 171-190.

[15] Jaynes, E.T., 2003. Probability Theory: The Logic of Science, Cambridge University Press, Cambridge, UK.

[16] Skilling, J., 1989. Maximum Entropy and Bayesian Methods, Kluwer, Dordrecht.

[17] Caticha, A., 2004. Relative entropy and inductive inference, in Bayesian Inference and Maximum Entropy Methods in Science and Engineer, G. Erickson and Y. Zhai (Eds.), Vol. 707, AIP, New York.

[18] Johnson, V.E., and Albert, J.H., 1999. Ordinal Data Modeling, Springer, New York.

[19] Lin, C.W., Shieh, C.L., Yuan, B.D., Shieh, Y.C., Liu, S.H., and Lee, S.Y., 2003, Impact of Chi-Chi earthquake on the occurrence of landslides and debris flows: example from the Chenyulan River watershed, Nantou, Taiwan, Engineering Geology, 71, 49-61.

[20] Chen, J.D., Wu, H.L., and Chen, L.J., 1997. A comprehensive debris flow hazard mitigateion program in Taiwan, in Proceedings of 1st International Conference on Debris-flow Hazards Mitigation: Mechanics, Prediction, and Assessment, C.L. Chen (Ed.), 93-102, Am. Soc. Civil Engineers, New York.

[21] Jan, C.D., and Chen, C.L., 2005. Debris flows caused by Typhoon Herb in Taiwan, in Debris-flow hazards and related phenomena, M. Jakob and O. Hungr (Eds.), 539-563, Springer, New York.

[22] Chen, H., and Su, D.Y., 2001. Geological factors for hazardous debris flows in Hoser, central Taiwan, Environmental Geology, 40, 1114-1124.

[23] Chen, J.D., Su, R.R., and Wu, H.L., 2000. Hydrometeorological and site factors contributing to disastrous debris flows in Taiwan, in Debris-flow Hazards Mitigation: Mechanics, Prediction, and Assessment, G.F. Wieczorek and N.D. Naeser (Eds.), 583-592, A.A. Balkema, Rotterdam, Netherlands.

[24] Lin, M.L., and Jeng, F.S., 2000. Characteristics of hazards induced by extremely heavy rainfall in Central Taiwan: Typhoon Herb, Engineering Geology, 58, 191-207.